

Simulation

10

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Simulation Techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- ⦿ Recognise a problem of Simulation.
- ⦿ Solve problems of Simulation using Monte Carlo Simulation technique.

Simulation is the most important technique used in analyzing complex real world problems which cannot be represented by a mathematical model due to their stochastic nature, complexity in their formulation and involvement of too many unknown variables. It has become an important tool for tackling the complicated managerial decision making problems. It determines the effect of a number of alternate policies without disturbing the real system. Simulation is a modeling and analysis tool widely used for the purpose of designing, planning, and control of manufacturing systems. Simulation in general is to pretend that one deals with a real thing while really working with an imitation. In operations research, the imitation is a computer model of the simulated reality. The task of executing simulations provides insight and a deep understanding of physical processes that are being modeled.

Simulation is generally referred to as computer simulation, which simulates the operation of a manufacturing system. A computer simulation or a computer model is a computer program which attempts to simulate an abstract model of a particular system. Computer simulation was developed hand-in-hand with the rapid growth of the computer, following its first large-scale deployment during the Manhattan Project in World War II to model the process of nuclear detonation. The job was done by two mathematicians John Von Newman and Stanislaw Ulam. Computer simulation is often used as an adjunct to, or substitution for, modeling systems for which simple closed form analytic solutions are not possible. There are many different types of computer simulation; the common feature they all share is the attempt to generate a sample of representative scenarios for a model in which a complete enumeration of all possible states of the model would be prohibitive or impossible.

With a computer simulation model, a manager or system analyst is able to observe the behaviour of a process without the necessity of experimenting with the actual system. In order to evaluate the system's performance given various disturbances, or to identify the bottlenecks, they may try out different manufacturing runs, new operational conditions, new equipment layouts or different cycle times.

A simple example of a simulation involves the tossing of a ball into the air. The ball can be said to "simulate" a missile, for instance. That is, by experimenting with throwing balls starting at different initial heights and initial velocity vectors, it can be said that we are simulating the trajectory of a missile. This kind of simulation is known as analog simulation since it involves a physical model of a ball. A flight simulator on a PC is a computer model of some aspects of the flight: it shows on the screen the controls and what the "pilot" (the youngster who operates it) is supposed to see from the "cockpit" (his armchair).

Definitions

Simulation can be defined as "A quantitative technique that uses a computerized symbolic model in order to represent actual decision making under uncertainty for determining alternative courses of action based upon facts and assumptions".

Few other definitions of Simulation are as follows –

- ⊙ Simulation is a representation of reality through the use of model or other device, which will react in the same manner as reality under a given set of conditions.
- ⊙ Simulation is the use of system model that has the designed characteristics of reality in order to produce the essence of actual operation.
- ⊙ According to Donald G. Malcolm - Simulation model may be defined as one which depicts the working of a large scale system consisting of men, machine, materials and information, operating over a period of time in a simulated environment of the actual real world conditions.
- ⊙ As per Naylor - Simulation is a numerical technique for conducting experiments on a computer, which involves certain type of mathematical and logical relationships necessary to describe the behavior and structure of a complex real world system over extended period of time.

Application of Simulation

- (i) Scheduling aircraft,
- (ii) Job-shop scheduling and personnel scheduling,
- (iii) Manpower-hiring decisions,
- (iv) Traffic light-timing,
- (v) Transport-scheduling,
- (vi) Evaluating alternative investment opportunities,
- (vii) Design of parking lots, harbour, and communication systems etc.

Advantages of Simulation

- (a) Enables to experiment and study complex interactions of a system (e.g. company operations, economic policies).
- (b) Possible to study the effects of organizational environment informational changes in the operations of a system (e.g. number of stocking points, industrial policies).
- (c) Better insight and understanding of a complex system resulting in indication for improvement.
- (d) Assists in teaching and training (management games).
- (e) New situations policies can be protested.
- (f) Probabilistic features can be easily incorporated.
- (g) A process can be studied in extended or compressed time.
- (h) Risks involved in experimenting with real problems can be eliminated.

Limitations of Simulation

- (a) Simulated results are not precise. Unlike mathematical models, it does not give optimum solutions. At times one may not be able to assess the extent of error in a simulated result.
- (b) Some situations are not amenable to simulation.

- (c) Simulation may be expensive needing advanced computer supports
- (d) Simulation by itself does not generate solutions, but only indicates a way of evaluating solutions.
- (e) It is often a long complicated process to develop a model.

Types and Models of Simulation

There are two types of Simulation.

- (a) Analogue Simulation: In this type, Reality is simulated in physical form. Examples of this type are as follows
 - ⊙ Children's Park in a city with various signals and crossing is the simulated model of city traffic.
 - ⊙ Planetarium is the simulated model of solar system.
- (b) Computer Simulation: For problems of complex Managerial Decision making, it is not possible to apply the method of Analogue Simulation. In this case, some mathematical model is formulated and subsequently solved using high speed computers.

Simulation Models are generally classified into four categories as described below.

- 1. Deterministic Models: In this case, the input and output variables are not random. Rather they are described by exact functional relationship.
- 2. Probabilistic Models: For such cases, method of random sampling is used. The technique used for solving these is known as Monte Carlo technique.
- 3. Static Models: These models do not take variable time into consideration.
- 4. Dynamic Models: These models deal with time varying interaction.

Steps involved in constructing and using a Simulation Model

All simulation projects follow the steps given below:

- (i) Define the problem
- (ii) Data collection
- (iii) Problem analysis
- (iv) Simulation model specification
- (v) Model programming
- (vi) Model validation
- (vii) Simulation experimentation
- (viii) Evaluation and interpretation of simulation results
- (ix) Report generation and plans for implementation of the results.

Monte Carlo Simulation

Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, and other forecasting models. It is a Simulation technique in

which statistical distribution functions are created using a series of random numbers. This method is generally used to solve problems which cannot be adequately represented by mathematical models or where solution of the problem is not possible by analytical method. This method yields a solution which is very close to the optimal but not the exact optimum one. But the solution given by this method converges to the optimum as the number of simulation trials tend to infinity. This technique has derived its name from the gambling establishments of Monte Carlo because the samples are selected in a purely random manner.

Following are the steps to solve a problem by Monte Carlo Simulation.

1. Clearly define the problem in order to determine the objectives and constraints.
2. Construct the appropriate model and decide which variables are to be included. Ensure that the model is a true representative of the real life problem.
3. Specify values of variables to be tested. Supply values of input parameters and measure the output values.
4. Collect the information required. Also determine the functional relationships and the types of probability distribution which are to be applied.
5. Define a coding system that will correlate the factors identified in step (1) with the random numbers that will be generated for the simulation.
6. Select a random number generator and create the random numbers to be used in the simulation.
7. Correlate the generated random numbers with the factors identified in steps (1) & (5) above.
8. Summarise and examine the results.
9. Evaluate the results of the simulation and select the best course of action.
10. Formulate proposals for advising the management on new policies and modify the model if necessary.

Random Numbers

Random Number is a number in a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence. Suppose we are interested in one digit numbers that is any number from the sequence 0, 1, 2, 3, 9, As there are ten numbers in all, by the above mentioned concept of random numbers we have to get equally likely number from the sequence i.e probability of generating any number from the sequence should be 1/10. Tables of random numbers are available in different books and literatures. Usually for Simulation on computers, random numbers are generated through a computer programme itself. Such random numbers are called **Pseudo Random Numbers**. A sequence of Pseudo Random Number is not truly random because it is generated using a deterministic mathematical process. But it satisfies the statistical test for randomness.

Process of Pseudo Random Number Generation is described in brief in the following lines. The process starts with an initial number called *Seed* which generates the second number and from that a third number and so on. Of the various available recursive procedures, the most common one is **Congruence Method** or the **Residue Method**. It is described with the help of the expression

$$r_{i+1} = (a.r_i + b) \text{ (modulo } m)$$

where a, b and m are constants and r_i as well as r_{i+1} are i th and $(i+1)$ th Random numbers. The expression implies multiplication of 'a' by r_i and subsequent addition of b with it and division of the result by m. Then r_{i+1} is the remainder or residue. To begin the process of random number generation along with the values of a, b and m the value of r_0 should also be known. It may be any random number and is called **Seed**. The expression above is called **Mixed type Congruential Random Number Generator**. If a = 1 then the expression becomes $r_{i+1} = (r_i +$

b) (modulo m) which is called **Additive type Congruential Random Number Generator**. Similarly when $b = 0$ then the expression for **Multiplicative type Congruential Random Number Generator** is obtained as $r_{i+1} = (a.r_i)$ (modulo m)

In the following illustration, the procedure of random number generation is further clarified.

Illustration 1

Generate a string of 12 pseudo random numbers by Mixed type Residue Method when it is given that the values of the constants are $a = 4$, $b = 7$, $m = 9$ and the Seed is 1.

Solution:

The expression for Random Number Generator by Mixed type Residue Method is given as follows –

$$r_{i+1} = (a.r_i + b) \text{ (modulo } m) \text{ where } a = 4, b = 7, m = 9 \text{ and Seed } = r_0 = 1$$

Calculations for Random Number generation by Mixed type Residue Method

r_i	$r_{i+1} = (a.r_i + b) \text{ (modulo } m)$	Result with Residue
r_1	$r_1 = (a.r_0 + b)/m = (4.1 + 7)/9 = 11/9$	1 with residue 2
r_2	$r_2 = (a.r_1 + b)/m = (4.2 + 7)/9 = 15/9$	1 with residue 6
r_3	$r_3 = (a.r_2 + b)/m = (4.6 + 7)/9 = 31/9$	3 with residue 4
r_4	$r_4 = (a.r_3 + b)/m = (4.4 + 7)/9 = 23/9$	2 with residue 5
r_5	$r_5 = (a.r_4 + b)/m = (4.5 + 7)/9 = 27/9$	3 with residue 0
r_6	$r_6 = (a.r_5 + b)/m = (4.0 + 7)/9 = 7/9$	0 with residue 7
r_7	$r_7 = (a.r_6 + b)/m = (4.7 + 7)/9 = 35/9$	3 with residue 8
r_8	$r_8 = (a.r_7 + b)/m = (4.8 + 7)/9 = 39/9$	4 with residue 3
r_9	$r_9 = (a.r_8 + b)/m = (4.3 + 7)/9 = 19/9$	2 with residue 1
r_{10}	$r_{10} = (a.r_9 + b)/m = (4.1 + 7)/9 = 11/9$	1 with residue 2
r_{11}	$r_{11} = (a.r_{10} + b)/m = (4.2 + 7)/9 = 15/9$	1 with residue 6

So the required string of Pseudo Random Numbers – 1, 2, 6, 4, 5, 0, 7, 8, 3, 1, 2, 6,

Illustration 2

The past data of demand per week (in '00 kgs.) of a confectionery item is given below –

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the sequence of random numbers – 35, 52, 13, 90, 23, 73, 34, 57, 35, 83, 94, 56, 67, 66 generate the demand for the next 10 weeks. Also find out the average demand per week

Solution:

Table showing Random Number Range for Demand

Demand/ week	Frequency (f)	Probability ($p = f / \sum f$)	Cumulative Probability	Range
0	2	.04	.04	00-03
5	11	.22	.26	04-25
10	8	.16	.42	26-41
15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
Total	$\sum f = 50$	1.00		

Table showing Simulated values for the next ten weeks

Week	Random No.	Demand in '00 Kgs.
1	35	10
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
		Total = 120

Explanatory Note on the method of obtaining simulated demand :

35 is the first one of the given Random Nos. So it is used for Week 1. Also 35 lies within the Range 26-41 of the previous table. Again 10 is the demand / week for the range 26-41. Hence demand for week 1 is 10. Similarly, the demands for the other weeks are simulated.

Average demand per week = Total demand / No. of weeks = $120/10 = 12$ ('00) Kgs.

Illustration 3

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs ₹ 60 and is sold for ₹ 80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for ₹ 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No of copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate the demand for 20 time periods (years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14	02	93	99	18	71	37	30	12	10
88	13	00	57	69	32	18	08	92	73

Solution:

Table showing Range of Random Numbers

Demand (No. of copies)	Probability	Cumulative Probability	Random No. Range
15	.05	.05	00-04
16	.08	.13	05-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99
Total	1.00		

Given that Cost of a book = ₹ 60 and Selling Price = ₹ 80

So profit per sold book = 80 – 60 = ₹ 20

Also selling price of each unsold book = ₹ 30

Simulated demand for the next 20 years and corresponding calculation of number of unsold books for different inventory levels

Year	Random Number	Expected demand in Nos.	No. of books unsold if stock* is		
			16	17	18
1	14	17	-	-	1
2	02	15	1	2	3
3	93	20	-	-	-
4	99	22	-	-	-
5	18	17	-	-	1
6	71	18	-	-	-
7	37	18	-	-	-
8	30	17	-	-	1
9	12	16	-	1	2
10	10	16	-	1	2
11	88	20	-	-	-
12	13	17	-	-	1
13	00	15	1	2	3
14	57	18	-	-	-
15	69	18	-	-	-
16	32	17	-	-	1
17	18	17	-	-	1
18	08	16	-	1	2
19	92	20	-	-	-
20	73	18	-	-	-
Total			2	7	18

[*Note: Stock figures of 16, 17 & 18 are chosen based on the simulated demand figures.]

Computation of Profit

No. of Books in stock	No. of Books sold during 20 years	Profit (₹)	Average Profit (₹/Year)
15	$15 \times 20 = 300$	6000	₹ 300
16	$(16 \times 20) - 2 = 318$	6300 $(318 \times 20) - 2 \times 30$	₹ 315
17	$(17 \times 20) - 7 = 333$	6450 $(333 \times 20) - 7 \times 30$	₹ 322.5
18	$(18 \times 20) - 18 = 342$	6300 $(342 \times 20) - 18 \times 30$	₹ 315

As average profit is maximum (i.e ₹ 322.50) when there is a stock of 17 books, the optimal policy is to place an order of 17 books.

Illustration 4

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information

Weekly Receipts (₹)	Probability	Weekly Payments (₹)	Probability
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is Rs. 8000. What is the estimated balance at the end of the 12 week period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

Random Numbers

For Receipts	03	91	38	55	17	46	32	43	69	72	24	22
For Payments	61	96	30	32	03	88	48	28	88	18	71	99

Solution:

Table showing Range of Random Numbers for Receipts and Payments

Receipt (₹)	Probability	Cumulative Probability	Range	Payment (₹)	Probability	Cumulative Probability	Range
3000	0.20	0.20	00-19	4000	0.30	0.30	00-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulated values of Receipts & Payments for the next 12 weeks and Calculation of week end Balances

Week	Random No. for Receipts	Expected Receipts (₹)	Random No. for Payments	Expected Payments (₹)	End of week Balance (₹)
Opening balance					8000
1	03	3000	61	6000	5000
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	(3000)
Total					45000

[**N.B** - End of week Balance for a particular week = End of week Balance for the previous week + Receipt during the week – Payment made in the week]

Estimated balance at the end of 12 th week	=	₹ (3,000)
Highest weekly balance during the quarter	=	₹ 7,000
Average weekly balance for the quarter	=	45,000/12 = ₹ 3750

Illustration 5

Patients arriving at a village dispensary are treated by a doctor on a first-come-first-served basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the system and determine the average time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle.

Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times and between 15 and 40 to estimate the service times required by the patients.

Series 1	07	21	12	80	08	03	32	65	43	74
Series 2	23	37	16	28	30	18	25	34	19	21

Solution:**Simulated Inter-arrival & Service Times and Calculation of Patient's Waiting time & Doctor's Idle time**

Patient No.	Inter arrival time Random No. (minutes)	Entry time in to the queue	Service Time Random No. (minutes)	Service Start time	Service End time	Waiting time of patient (minutes)	Idle time of doctor (minutes)
1	07	8.07 A.M	23	8.07 A.M	8.30 A.M	-	7
2	21	8.28 A.M	37	8.30 A.M	9.07 A.M	2	-
3	12	8.40 A.M	16	9.07 A.M	9.23 A.M	27	-
4	80	10.00 A.M	28	10.00 A.M	10.28 A.M	-	37
5	08	10.08 A.M	30	10.28 A.M	10.58 A.M	20	-
6	03	10.11 A.M	18	10.58 A.M	11.16 A.M	47	-
7	32	10.43 A.M	25	11.16 A.M	11.41 A.M	33	-
8	65	11.48 A.M	34	11.48 A.M	12.22 P.M	-	7
9	43	12.31 P.M	19	12.31 P.M	12.50 P.M	-	9
10	74	1.45 P.M	21	1.45 P.M	2.06 P.M	-	55
Total						129	115

[N.B – The above table is prepared on the basis of the assumption that the dispensary opened at 8.00 A.M]

Average time a patient has to be in the queue for getting service = $129/10 = 12.9$ minutes

Doctor is there in the dispensary from 8.00 A.M to 2.06 P.M ie for 6 hours & 6 minutes = 366 minutes.

During this period he is idle for 115 minutes. So proportion of time the doctor is idle = $115/366 = 0.314$

Illustration 6

A businessman is considering taking over a certain new business. Based on past information and his own knowledge of the business, he works out the probability distribution of the monthly costs and sales revenues, as given here:

Cost (in ₹)	Probability	Sales Revenue (₹)	Probability
17000	0.10	19000	0.10
18000	0.10	20000	0.10
19000	0.40	21000	0.20
20000	0.20	22000	0.40
21000	0.20	23000	0.15
		24000	0.05

Use the following sequences of random numbers for estimating costs and revenues. Obtain the probability distribution of the monthly net revenue.

Sequence 1	82	84	28	82	36	92	73	91	63	29
(for Cost)	27	26	92	63	83	02	10	39	10	10

Sequence 2	39	72	38	29	71	83	19	72	92	59
(for Revenue)	49	39	72	94	04	92	72	18	09	00

Solution:

Table showing Range of Random Nos. for Cost and Revenue

Cost (₹)	Probability	Cumulative Probability	Random No. Range	Revenue (₹)	Probability	Cumulative Probability	Random No. Range
17000	0.1	0.1	00 - 09	19000	0.1	0.1	00 - 09
18000	0.1	0.2	10 - 19	20000	0.1	0.2	10 - 19
19000	0.4	0.6	20 - 59	21000	0.2	0.4	20 - 39
20000	0.2	0.8	60 - 79	22000	0.4	0.8	40 - 79
21000	0.2	1.0	80 - 99	23000	0.15	0.95	80 - 94
				24000	0.05	1.00	95 - 99

Table showing simulated values of Cost and Sales for the next 20 months

Month	Random No. for Cost	Cost (₹)	Random No. for Sales	Sales (₹)	Monthly Net Revenue (₹)
1	82	21000	39	21000	0
2	84	21000	72	22000	1000
3	28	19000	38	21000	2000
4	82	21000	29	21000	0
5	36	19000	71	22000	3000
6	92	21000	83	23000	2000
7	73	20000	19	20000	0
8	91	21000	72	22000	1000
9	63	20000	92	23000	3000
10	29	19000	59	22000	3000
11	27	19000	49	22000	3000
12	26	19000	39	21000	2000
13	92	21000	72	22000	1000
14	63	20000	94	23000	3000
15	83	21000	04	19000	(2000)
16	02	17000	92	23000	6000
17	10	18000	72	22000	4000
18	39	19000	18	20000	1000
19	10	18000	09	19000	1000
20	10	18000	00	19000	1000

Probability Distribution for Monthly Net Revenue

Net Revenue (₹)	(2000)	0	1000	2000	3000	4000	6000	Total
Probability	1/20	3/20	6/20	3/20	5/20	1/20	1/20	1

EXERCISE**A. Theoretical Questions:****⊙ Multiple Choice Questions**

1. Which of the following is first step for performing Simulation analysis?
 - (a) Choose input variables.
 - (b) Create entities for the simulation process.
 - (c) Prepare a problem statement.
 - (d) Determine the output variables.

2. Which of the following are the advantages of using Modelling and Simulation?
 - (a) Easy to understand.
 - (b) Easy to test.
 - (c) Easy to upgrade.
 - (d) All of the above.

3. Which one of the following is not an application area of Modelling and Simulation?
 - (a) Military applications
 - (b) Designing semiconductors
 - (c) Telecommunications
 - (d) Food industry

4. Which of the following is the first step for developing the Simulation Model?
 - (a) Design the problem
 - (b) Identify the problem.
 - (c) Collect and start processing the system data
 - (d) Develop the model using Network diagram.

5. Simulation is the process of using a model to study the performance of a system.
 - (a) Agreeable
 - (b) Not agreeable.
 - (c) Partly agreeable
 - (d) Cannot comment.

6. Disadvantage of using Modelling and Simulation lies in the statement –
 - (a) Simulation requires manpower and it is a time consuming process.
 - (b) Simulation results are difficult to translate and only experts can understand it.

- (c) Simulation is an expensive process.
 - (d) All of the above.
7. Monte Carlo Simulation gets its name from which of the following?
- (a) Data collection.
 - (b) Model formulation
 - (c) Random number assignment
 - (d) Analysis
8. Select the valid reasons for using Simulation.
- (a) Relationship between the variables is non-linear.
 - (b) Optimized solutions are obtained.
 - (c) Conduct experiment without disrupting the real system.
 - (d) Both (a) and (c)
9. Simulation uses logical relationship and mathematical expressions of the –
- (a) Real system
 - (b) Computer model
 - (c) Performance measures
 - (d) Inferences
10. While assigning random numbers in Monte Carlo Simulation, it is –
- (a) Not necessary to assign the exact range of random number interval as the probability.
 - (b) Necessary to develop a cumulative probability distribution.
 - (c) Necessary to assign the particular appropriate random numbers.
 - (d) All of the above.
11. Random numbers are used –
- (a) To give random outcomes.
 - (b) To describe the uncertainty of the input values.
 - (c) To assign values to the parameters.
 - (d) To change the problem solution.
12. Simulation can keep track of several different kinds of information, including orders, inventory, financial planning
- (a) Correct
 - (b) Partly correct

- (c) Totally incorrect
 - (d) None of the above
13. Monte Carlo Simulation ensures that –
- (a) The simulated probability distribution will be the same as the actual probability distribution.
 - (b) Only one uncertain decision can be taken in any simulation model.
 - (c) Probabilities will have at most two decimal place values.
 - (d) Each one of the above statement is true.
14. What should a Project Manager do when his preliminary task estimate differs from Monte Carlo Simulation?
- (a) Perform more Monte Carlo Simulation runs.
 - (b) Use this information to assess project uncertainty and risk and to review and possibly change crucial variables.
 - (c) Scrap the project as untenable.
 - (d) Discuss these differences at future project team meetings.
15. What can be expected at the end of a Monte Carlo Simulation exercise?
- (a) A conservative outcome.
 - (b) A decision backed by number that makes the decision maker confident about moving forward.
 - (c) All outcomes from all possible probabilities exercise.
 - (d) None of the above.
16. Analytical results are taken into consideration before a simulation study so as to -
- (a) Identify suitable values of the system parameter
 - (b) Determine the optimal decision.
 - (c) Identify the suitable values of the decision variables for the specific choice of the system.
 - (d) All of the above.
17. Which of the following statements are applicable for Pseudo Random Numbers?
- (a) They are numbers similar to Random Numbers.
 - (b) They are generated by using Remainder Method.
 - (c) They satisfy the statistical test for randomness.
 - (d) All of the above.
18. Large complicated simulation models are appreciated because –
- (a) Their average costs are not well defined
 - (b) It is difficult to create the appropriate events

- (c) Both (a) and (b) are true.
 - (d) None of the above is true.
19. Simulation should not be applied in all cases because it –
- (a) Requires considerable talent for model building and extensive computer programming efforts.
 - (b) Consumes too much computer time.
 - (c) Provides at best an approximate solution to the problem.
 - (d) All of the above.
20. Simulation is defined as –
- (a) A technique that uses computers.
 - (b) An approach for reproducing the processes by which events by chance and changes are created by computers.
 - (c) A procedure for testing and experimenting on models to answer what if, then so and so type questions.
 - (d) All of the above.
21. Few causes of simulation analysis failure are –
- (a) Inadequate level of user participation.
 - (b) Inappropriate levels of detail.
 - (c) Incomplete mix of essential skills.
 - (d) All of the above.
22. To make simulation more popular –
- (a) Large cost overruns need to be avoided.
 - (b) Prolonged delays need to be avoided.
 - (c) User dissatisfaction with simulation results need to be avoided.
 - (d) All of the above.
23. The important step required for simulation approach in solving a problem is –
- (a) Test and validate the model.
 - (b) Design the experiment.
 - (c) Conduct the experiment.
 - (d) All of the above.
24. The general purpose system simulation language –
- (a) Requires program writing
 - (b) Does not require program writing.

- (c) Requires predefined coding forms.
- (d) Needs a set of equations to describe a system.
25. An advantage of simulation as opposed to optimization is that –
- (a) Several options of measure of performance can be examined.
- (b) Complex real life problems can be studied.
- (c) It is applicable to cases where there is randomness in a system
- (d) All of the above.
26. The purpose of using simulation technique is to –
- (a) Imitate a real world situation.
- (b) Understand properties and operating characteristics of complex real life problems.
- (c) Both (a) and (b) above.
- (d) Only (b) but not (a).
27. As simulation is not an analytical model, its solution should be viewed as –
- (a) Exact
- (b) Unrealistic
- (c) Approximation.
- (d) Simplified
28. All of the following are advantages of simulation except –
- (a) Facilitates in finding the optimal solution.
- (b) It is a low cost process.
- (c) It deals with playing Games.
- (d) Time compression is an issue with it.
29. The drive up window of a fast food operation was being studied using simulation for a variety of operating characteristics. As part of the study data was collected on Order Processing Time as given in the following table. Using the first two digits of the Random Numbers, determine the processing time that would be used to simulate the fifth sample.

Processing time (Minutes)	1	2	3	4						
Relative Frequency	0.30	0.45	0.20	0.05						
Customer	1	2	3	4	5	6	7	8	9	10
Random No.	1048	2236	2413	4216	3757	1501	4657	4836	9309	3997

- (a) 2 minutes

- (b) 4 minutes
- (c) 1 minute
- (d) 3 minutes

30. The drive up window of a fast food centre was being studied using simulation for a variety of operating characteristics. As part of the study data was collected on Customer Arrivals as given in the following table. Using expected value calculations determine the expected time between customer arrivals.

Inter arrival time (Minutes)	0.5	1.0	2.0	3.0	4.0	5.0	6.0
Probability	0.10	0.25	0.20	0.30	0.05	0.05	0.05

- (a) 2.35 minutes
- (b) 2.00 minutes
- (c) 2.70 minutes
- (d) 1.65 minutes

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	d	d	b	a	d	c	d	a	b	b	a	a	b	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
c	d	b	d	d	d	d	d	b	d	c	c	b	a	a

⦿ **Fill in the blanks**

1. Congruence Method is used to generate _____ numbers.
2. Monte Carlo Simulation is also known as _____ Simulation.
3. In _____ model of simulation, input and output variables are not random.
4. The expression $[r_{i+1} = (a \cdot r_i) \pmod{m}]$ is for _____ type Congruence Random Number generator.
5. Planetarium is an example of _____ Simulation for the solar system.
6. _____ simulation is used for solving complex problem of managerial decision making.
7. The ever first large scale deployment of simulation was done during World War II to model the process of _____
8. Aircraft _____ is an application area of simulation.
9. _____ types of Congruence Methods are used to generate Pseudo Random Numbers.
10. _____ technique can be used to solve Queuing problems.

Answers:

1.	Pseudo Random	2.	Probability
3.	Deterministic	4.	Multiplicative
5.	Analogue	6.	Computer
7.	Nuclear detonation	8.	Scheduling
9.	Three	10.	Monte Carlo

⊙ **State True or False**

- Simulation is a modelling and analysis tool.
- Probability of occurrence of any single digit Random number is 1/10.
- Simulation models are designed to generate optimal solutions which can then be applied to real world situations.
- One of the major advantage of simulation is time compression i.e the ability to study in a relatively short period of the activities which normally take months if considered in real world terms.
- While it is powerful, Simulation is not considered to be a flexible quantitative analysis tool.
- Simulation can use any probability distribution that the user defines. It does not require standard distributions.
- Simulation of a business or process is generally performed by building a mathematical model to represent the process or system.
- A major advantage of using simulation techniques is to be able to study the interactive effect of individual components or variables.
- To simulate is to try to duplicate the features, appearance and characteristics of a real world system.
- Simulations are called Models because they make use of mathematics.
- Simulations are very flexible. Thus its solutions and inferences are transferable to other problems.
- If the Monte Carlo Simulation is used then it is expected to get same result for each set of random numbers used.
- Simulation is a numerical technique for conducting experiments on a computer which involves certain type of mathematical and logical relationship necessary to describe the behaviour of a complex real world system.
- Calculation of cumulative probability is a step in Monte Carlo model.
- “Some situations are not amenable to simulation” – is a limitation of simulation.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	T	F	T	F	T	T	T	T	F	F	F	T	T	T

⊙ Short essay type questions

1. “Children's Park in a city” is the example of what type of simulation? Explain.
2. “Simulation generates a near optimal solution of a real life problem related to any process”– Discuss
3. Name five application areas of Simulation.

⊙ Essay type questions

1. What are the advantages of Simulation?
2. Describe the limitations of Simulation.
3. Write short notes on the methods of generation of Pseudo Random Numbers.

B. Numerical Questions

⊙ Comprehensive Numerical Problems

1. An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below

Production/Day	Prob.	Production/Day	Prob.
95	0.03	101	0.15
96	0.05	102	0.10
97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20		
		Total	1.00

Finished cars are transported across the bay, at the end of each day, by ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped, and what will be the average number of empty space on the boat? Random numbers are given below:

20, 63, 46, 16, 45, 41, 44, 66, 87, 26, 78, 40, 29, 92, 21

2. After observing heavy congestion of customers over a period of time in a petrol station, Mr. Petro has decided to set up a petrol pump facility on his own in a nearby site. He has compiled statistics relating to the potential customer arrival pattern and service pattern as given below. He has also decided to evaluate the operations by using the simulation technique

Arrivals		Services	
Inter-arrival time (minutes)	Probability	Inter-arrival time (minutes)	Probability
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume:

- (i) The clock starts at 8:00 hours
- (ii) Only one pump is set up.
- (iii) The following 12 Random Numbers are to be used to depict the customer arrival pattern: 78, 26, 94, 08, 46, 63, 18, 35, 59, 12, 97 and 82.
- (iv) The following 12 Random Numbers are to be used to depict the service pattern: 44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37

You are required to find out the

- (i) probability of the pump being idle, and
 - (ii) Average time spent by a customer waiting in queue.
3. The Tit-Fit Scientific Laboratories is engaged in producing different types of high class equipment for use in science laboratories. The company has two different assembly lines to produce its most popular product 'Pressure'. The processing time for each of the assembly lines is regarded as a random variable and is described by the following distributions.

Process Time (minutes)	Assembly A1	Assembly A2
10	0.10	0.20
11	0.15	0.40
12	0.40	0.20
13	0.25	0.15
14	0.10	0.05

Using the following random numbers, generate data on the process times for 15 units of the item and compute the expected process time for the product. For the purpose, read the numbers vertically taking the first two digits for the processing time on assembly A1 and the last two digits for processing time on assembly A2.

4134	8343	3602	7505	7428
7476	1183	9445	0089	3424
4943	1915	5415	0880	9309

4. A book store wishes to carry 'Ramayana' in stock. Demand is probabilistic and replenishment of stock takes 2 days (i.e. if an order is placed on March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given:

Demand (daily)	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of ₹ 10 per order. The store also incurs a carrying cost of ₹ 0.50 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day.

The manager of the bookstore wishes to compare two options for his inventory decision.

- A. Order 5 books when the inventory at the beginning of the day plus order outstanding is less than 8 books.
- B. Order 8 books when the inventory at the beginning of the day plus order outstanding is less than 8.

Currently (beginning 1st day) the store has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day.

Using Monte-Carlo Simulation for 10 cycles, recommend, which option the manager, should choose. The two digit random numbers are given below:

89 34 70 63 61 81 39 16 13 73

Answers:

1.	Average number of cars waiting to be shipped = 0.4, Average number of empty spaces = 1.2
2.	Probability of idle time of the petrol station = 0.14, Average time spent by a customer = 11.67 mins.
3.	Expected process time for the product = 23.27 minutes.
4.	Option B is better because it has lower inventory cost.

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